

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS  
APPLICATIONS  
UNIT 3

**SOLUTIONS**

Section Two:  
Calculator-assumed

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 7 | 7 | 50 | 51 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 (7 marks)

The graph below represents a network of distribution centres. Each edge weight is the cost in dollars to transport a parcel between adjacent centres (the vertices).

<EFOFEX>
id:fxd{f4975ce9-1086-4da4-9bdd-a2f74fac6595}

FXData:
</EFOFEX>

(a) Determine the minimum cost to transport a parcel from to and state the path that should be used to achieve this minimum. (3 marks)

|  |
| --- |
| Solution |
| Dijkstra's algorithm (not specified in course - vertex, min. distance from pairs):  Hence minimum cost is using path . |
| Specific behaviours |
| ✓ evidence of method (algorithm, listing of trials, etc.)  ü correct minimum cost  ü correct path |

(b) A new route is proposed between centres and which will reduce the minimum cost to transport a parcel from to by . Determine the cost to transport a parcel between centres and . (2 marks)

|  |
| --- |
| Solution |
| Hence cost is . |
| Specific behaviours |
| ✓ indicates appropriate method  ü correct cost |

(c) A parcel is transported along a route that is a cycle of edges in the graph. Determine the maximum possible transport cost and describe the corresponding cycle. (2 marks)

|  |
| --- |
| Solution |
| Cycle for maximum cost is and cost is .  (*Also etc*) |
| Specific behaviours |
| ✓ correctly describes a cycle  ü correct cost |

Question 9 (8 marks)

A random sample of adults who were not working and not seeking work were recently asked for the main reason that they were not looking for work. The responses, categorised by the sex of the adult and their main reason, are summarised in the table below.

|  |  |  |
| --- | --- | --- |
| Reason | Female | Male |
| Have job starting soon |  |  |
| Injury or disability |  |  |
| Other |  |  |

(a) How many adults gave a response? (1 mark)

|  |
| --- |
| Solution |
| adults. |
| Specific behaviours |
| ✓ correct total |

(b) What percentage of the males gave injury or disability as their main reason? (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates use of correct figures  ü any percentage that rounds to |

(c) Construct a table showing column percentages for the above data, rounding entries to the nearest whole number. (3 marks)

|  |
| --- |
| Solution |
| |  |  |  | | --- | --- | --- | | Reason | Female (%) | Male (%) | | Have job starting soon |  |  | | Injury or disability |  |  | | Other |  |  | |
| Specific behaviours |
| ✓ neat table with row and column headings  ü both columns add to  ü both columns correct |

(d) Discuss whether the data from the survey suggests the presence of an association between the variables sex and reason. (2 marks)

|  |
| --- |
| Solution |
| The data does not suggest an association exists between the variables as the pairs of percentages in each row (i.e., for each reason) are very similar for males and females. |
| Specific behaviours |
| ✓ states no association exists  ü explains using similarity in percentages across reason categories |

Question 10 (7 marks)

(a) The monthly units of electricity consumed by each apartment in a building was strongly associated with the average monthly maximum temperature, C. The least-squares line for the variables was .

(i) Predict the units of electricity consumed by an apartment in a month when the average monthly maximum temperature was C. (1 mark)

|  |
| --- |
| Solution |
| units. |
| Specific behaviours |
| ✓ correct prediction |

(ii) In a month when the average monthly maximum temperature was C, an apartment consumed units of electricity. Calculate the residual for this data point. (2 marks)

|  |
| --- |
| Solution |
| Residual: units. |
| Specific behaviours |
| ✓ indicates correct  ü correct residual |

(b) In a government study, the correlation coefficient for the association between age and superannuation balance for employed adults was found to be . What percentage of the variation in superannuation balance for employed adults is unexplained by their variation in age? (2 marks)

|  |
| --- |
| Solution |
| Since of the variation is explained, then is unexplained. |
| Specific behaviours |
| ✓ calculates coefficient of determination  ü correct percentage |

(c) After measuring the age and hearing acuity of a group of pensioners, a researcher observed a negative linear association between the variables and found that of the variation in hearing acuity can be explained by the variation in age. Determine the correlation coefficient for the association. (2 marks)

|  |
| --- |
| Solution |
| Since association is negative, then . |
| Specific behaviours |
| ✓ indicates square root of coefficient of determination  ü correct correlation coefficient |

Question 11 (7 marks)

Angie had a bank account that paid no interest. At the start of the year her account balance was , and at the end of the first week and every week thereafter she withdrew .

(a) Calculate the balance of Angie's account after weeks. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct balance |

Let the balance in Angie's account at the end of the week be .

(b) Deduce a rule for and hence determine the balance of Angie's account after weeks. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct rule (accept recursive or term)  ü correct balance |

(c) For how many weeks was Angie able to withdraw ? (1 mark)

|  |
| --- |
| Solution |
| weeks, since . |
| Specific behaviours |
| ✓ correct number of weeks |

In the same year, Angie's friend Brynn had a similar account. The balance of his account at the end of the week was given by the recurrence relation .

(d) Determine the balance of Brynn's account after weeks. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct balance |

(e) At the end of one week during the year, the balance of Angie's account was identical to that of Brynn's. Determine which week this was and the balance of both accounts at that time. (2 marks)

|  |
| --- |
| Solution |
| At the end of the week, when their balances were . |
| Specific behaviours |
| ✓ correct week  ü correct balance |

Question 12 (13 marks)

The table below shows the life expectancy, in years, of males and females in nine countries in Oceania.

|  |  |  |
| --- | --- | --- |
| Country | Male () | Female () |
| Fiji |  |  |
| Guam |  |  |
| Kiribati |  |  |
| Palau |  |  |
| Papua New Guinea |  |  |
| Samoa |  |  |
| Solomon Islands |  |  |
| Tuvalu |  |  |
| Vanuatu |  |  |

(a) On the scatterplot below, plot the three missing data points from the table. (2 marks)

<EFOFEX>
id:fxd{ed95d923-2029-483a-848f-123a1e06496d}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ at least one point correctly plotted  ü all points correctly plotted |

(b) Determine the coefficient of determination between the variables and interpret its value in the context of the question. (2 marks)

|  |
| --- |
| Solution |
| of the variation in the female life expectancy can be explained by the variation in the male life expectancy. |
| Specific behaviours |
| ✓ correct coefficient (to at least dp)  ü correct interpretation |

(c) State the correlation coefficient between the variables and use its value to comment on the strength of the linear association between male and female life expectancy for these countries. (2 marks)

|  |
| --- |
| Solution |
| The linear association between the variables is strong. |
| Specific behaviours |
| ✓ correct value of  ü states association is strong |

(d) Determine the equation of the least-squares line to model the relationship between the variables and draw this line on the scatterplot. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct equation (coefficients to at least dp)  ü any reasonable line of best fit  ü ruled lined through & |

(e) The life expectancy of a male from New Zealand is . Predict, to the nearest year, the life expectancy of a female from the same country and comment on any factors that affect the validity of your prediction. (2 marks)

|  |
| --- |
| Solution |
| Despite the strong correlation, this prediction involves extrapolation and should be treated with caution. |
| Specific behaviours |
| ✓ prediction  ü notes dangers of extrapolation |

(f) The life expectancy of a male from the Marshall Islands is . Predict, to the nearest year, the life expectancy of a female from the same country and comment on any factors that affect the validity of your prediction. (2 marks)

|  |
| --- |
| Solution |
| With strong correlation and the age lying within the range of data, it is reasonable to assume the prediction is valid. |
| Specific behaviours |
| ✓ prediction  ü notes valid, with at least one reason |

Question 13 (8 marks)

The cooling system for a mobile cool room has just been turned on. The temperature C inside the cool room, hours later, is modelled by the linear recurrence relation

(a) Complete the table of temperatures below. (2 marks)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| (C) |  |  |  |  |  |  |  |

(b) Add a scale to the vertical axis below and then plot the temperature inside the cool room every hour. (3 marks)

|  |
| --- |
| Solutions (a) & (b) |
| See table and graph |
| Specific behaviours |
| ✓ at least correct table entries  ü all correct table entries  ✓ adds scale to vertical axis  ü at least points correctly plotted  ü all points correct, no line |

<EFOFEX>
id:fxd{d2d3c1e8-e9f2-4ddb-9886-3f781f0b965d}

FXData:

</EFOFEX>

(c) After how many hours does the model predict that the temperature inside the cool room will first reach within of its steady state? Justify your answer. (3 marks)

|  |
| --- |
| Solution |
| Using sequence, steady state temperature is C. Hence temperature must fall to C or below.  From sequence, and and so cool room will first reach required temperature after hours. |
| Specific behaviours |
| ✓ indicates steady state temperature  ü states correct number of hours  ü justifies time using terms of sequence |

Question 14 (7 marks)

A business bought a mainframe computer valued at . The value of the computer depreciated by each year.

(a) By how much did the value of the computer depreciate during the first year and what was its value one year after it was bought? (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ depreciation amount  ü value after one year |

(b) Deduce a recursive rule for , the value of the computer after years. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct multiplier  ü correct rule with initial term |

(c) Calculate the value of the computer after years. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value (accept reasonable rounding) |

(d) During which year does the value of the computer first depreciate by less than ? Justify your answer. (2 marks)

|  |
| --- |
| Solution |
| By observing terms of the sequence, the annual depreciation will first be less than during the year. |
| Specific behaviours |
| ✓ indicates appropriate reasoning  ü correct year, with reasoning |

Question 15 (10 marks)

An industrial chemist varied the amount of accelerant ( grams) used when making an epoxy resin and recorded the time taken ( seconds) for the resin to set. The results are shown below.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

The chemist suspected that a linear association might exist between the variables and calculated the correlation coefficient .

(a) After seeing this value of the correlation coefficient, the chemist said to their assistant "it looks like there is a strong and negative linear association between the variables". Explain this interpretation of the coefficient. (2 marks)

|  |
| --- |
| Solution |
| Since is close to , the strength of the linear association is strong.  Since , the direction of the linear association is negative. |
| Specific behaviours |
| ✓ explains strong  ü explains negative |

The chemist also noted that the least-squares line for the data was and used it to calculate nine residuals for the linear model as shown below, rounded to one decimal place.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |
| Residual |  |  |  |  |  |  |  |  |  |  |

(b) Show how the residual of was calculated and determine the residual associated with grams of accelerant. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates  ü subtracts from  ü calculates missing residual |

(c) Construct a residual plot for the data on the axes below. (3 marks)

<EFOFEX>
id:fxd{5eb71268-4a87-45cd-ac5f-e44cecaf953a}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ adds suitable scale to vertical axis  ü accurately plots at least points  ü accurately plots all points |

(d) Does the residual plot support the chemist's suspicions that a linear model fits the data? Explain your answer. (2 marks)

|  |
| --- |
| Solution |
| The residual plot does not support the linear model as a pattern is evident in the residuals. |
| Specific behaviours |
| ✓ states plot does not support linear model  ü states pattern evident in residuals |

Question 16 (6 marks)

The mass of a small puppy was measured as g when it was one week old. A week later its mass had increased by g.

(a) Assuming that the weekly mass of the puppy can be modelled by an arithmetic sequence, predict the mass of the puppy when it is weeks old. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates appropriate method  ü correct mass |

(b) Assuming that the weekly mass of the puppy can be modelled by a geometric sequence, predict the mass of the puppy when it is weeks old. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates common ratio  ü indicates appropriate method  ü correct mass |

(c) Comment on the usefulness of these models as the puppy gets older. (1 mark)

|  |
| --- |
| Solution |
| Not very useful, since both models have the mass of the puppy increasing for ever, yet all dogs reach their adult weight after a year or two. |
| Specific behaviours |
| ✓ sensible comment that notes models eventually not useful |

Question 17 (8 marks)

The annual number of mobile phone subscriptions and new cars sold in Queensland, as collated by a researcher, are shown in the table below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
| Subscriptions (, in millions) |  |  |  |  |  |  |  |
| New cars (, in thousands) |  |  |  |  |  |  |  |

The researcher wanted to identify whether new car sales in Queensland could be predicted from mobile phone subscriptions.

(a) Quantify the strength of the linear association between the variables and . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates |

(b) Determine the equation of the least-squares line that can be used to predict from .

(2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct coefficients  ü uses given variables |

(c) Use the least-squares line to predict the number of new car sales in another Australian state that had million mobile phone subscriptions, and comment, with reasons, on the validity of your prediction. (3 marks)

|  |
| --- |
| Solution |
| Sales predicted to be thousand cars.  This prediction is not valid as the line is derived from sales of cars and smartphones in Queensland. In another state, it is unlikely that the variables will have the same association as in Queensland. |
| Specific behaviours |
| ✓ correct prediction, noting units  ü states, with reasoning, that prediction not valid  ü supplies reason that prediction not valid |

(d) Describe a possible non-causal explanation for the observed association between mobile phone subscriptions and new cars sold. (2 marks)

|  |
| --- |
| Solution |
| The association is likely due to a common response to a third variable - the population of the state. As the population increases, so there will be more people to buy cars and mobile phone subscriptions. |
| Specific behaviours |
| ✓ identifies a confounding variable such as population (*do not accept time*)  ü explains common response to confounding variable |

Question 18 (9 marks)

The adjacency matrix for the connected planar graph is .

(a) Determine, with justification, the number of faces that has. (3 marks)

|  |
| --- |
| Alternative Solution |
| Vertices (number of matrix rows) and edges (sum of elements above matrix diagonal).  is a connected planar graph, so using Euler's formula then and so . has faces. |
| Specific behaviours |
| ✓ states number of vertices and edges  ü uses Euler's formula  ü states number of faces |

|  |
| --- |
| Solution |
| <EFOFEX> id:fxd{ab3d8132-1f03-40d8-a384-30f51955ac37}  FXData:  </EFOFEX>  has faces. |
| Specific behaviours |
| ✓ draws , correct and  ü draws in the plane  ü states number of faces |

(b) Use elements from the adjacency matrix to explain why is a simple graph. (3 marks)

|  |
| --- |
| Solution |
| Elements on the leading diagonal are all and so there are no loops.  All other elements in the matrix are or and so there are no multiple edges. |
| Specific behaviours |
| ✓ states no loops and no multiple edges  ü uses diagonal elements to justify no loops  ü uses other elements to justify no multiple edges |

(c) Ore's theorem states that a simple graph with vertices is Hamiltonian if, for every pair of distinct vertices and which are not adjacent, the sum of the degrees of and is greater than or equal to . Use Ore's theorem to show that is Hamiltonian. (3 marks)

|  |
| --- |
| Solution |
| has two pairs of vertices that are not adjacent:  and with degrees and respectively.  and with degrees and respectively.  For each pair, and is greater than , the number of vertices, and so is Hamiltonian.  (*NB Using adjacency matrix, non-adjacent pairs identified by elements not on leading diagonal, and degree is sum of elements in row.*) |
| Specific behaviours |
| ✓ identifies the two pairs of vertices that are not adjacent  ü states degrees of both pairs of vertices  ü shows sum of both pairs is equal to and states conclusion |

Question 19 (8 marks)

Zoe plans to invest in an account that pays interest of per month. At the end of each month, just after interest is added to the account, she will withdraw . The balance of her account, , after withdrawals can be modelled by the recurrence relation

(a) Determine the balance of the account after withdrawals have been made and describe how the balance has changed since the account was opened. (2 marks)

|  |
| --- |
| Solution |
| Account balance has increased by . |
| Specific behaviours |
| ✓ calculates  ü states balance has increased and amount of increase |

(b) Calculate the total withdrawn from the account after withdrawals, and hence show that the total interest paid into the account over this time is . (2 marks)

|  |
| --- |
| Solution |
| Total withdrawn: .  Hence . |
| Specific behaviours |
| ✓ calculates total withdrawn  ü shows how to derive total interest |

(c) The balance of Zoe's account will first exceed after the withdrawal. Determine the value of and state the balance of the account at this time. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of  ü value of |

(d) If, after making the withdrawal, Zoe then changed the amount she withdrew each month to , how this would change the way the account balance grew in the future? Justify your answer. (2 marks)

|  |
| --- |
| Solution |
| For this new sequence, the account balance doesn't grow but starts to decrease, as seen by , which is cents less than . |
| Specific behaviours |
| ✓ shows modified recursive rule  ü uses terms of new sequence to explain balance now decreases |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_